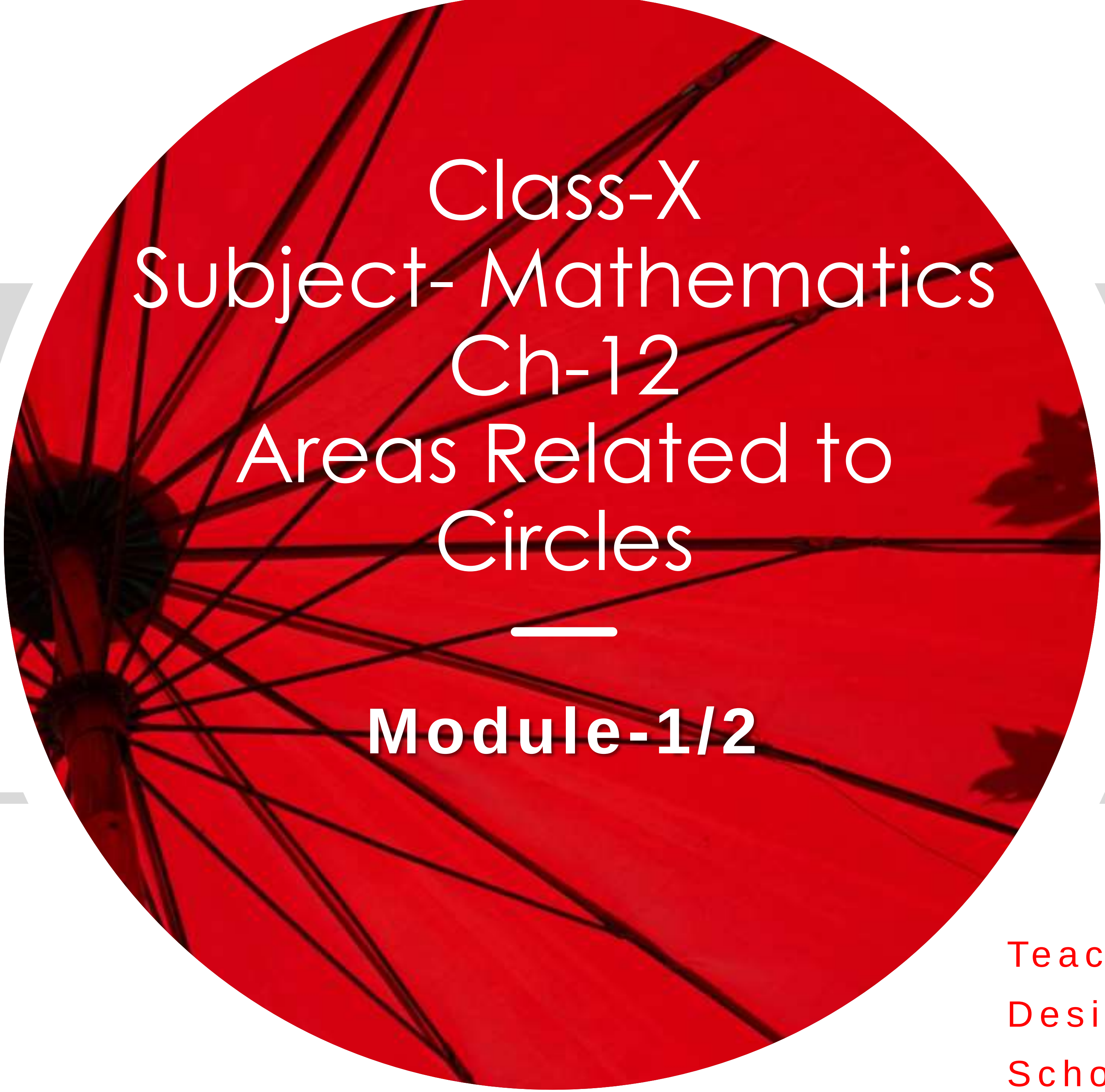

Class-X
Subject- Mathematics
Ch-12
Areas Related to Circles

Module-1/2

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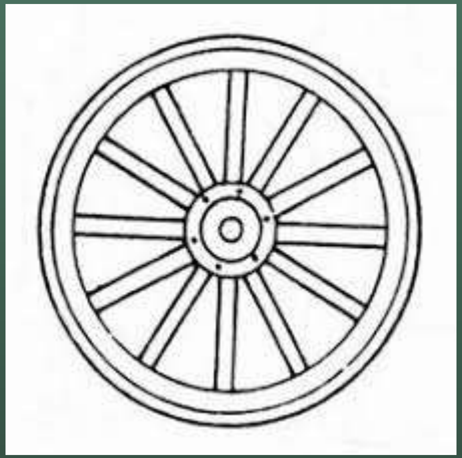
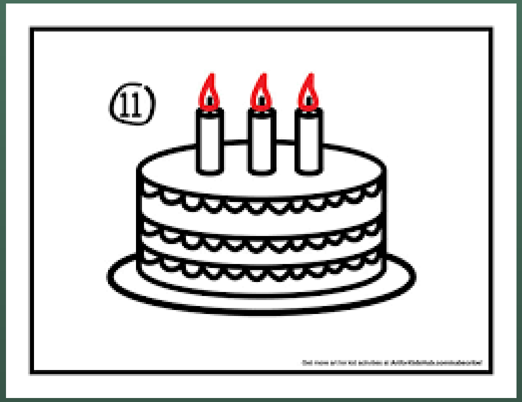
Designation: TGT
School: AECS Jaduguda

TOPICS

- Introduction
- Circumference and area of a circle.
- Areas of sector and segment of a circle.

Introduction

Many objects that we come across in our daily life are related to the circular shape in some form or the other. Cycle wheels, wheel barrow (thela), dartboard, round cake, papad, drain cover, various designs, bangles, brooches, circular paths, washers, flower beds, etc. are some examples of such objects. So, the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall begin our discussion with a review of the concepts of perimeter (circumference) and area of a circle and apply this knowledge in finding the areas of two special 'parts' of a circular region (or briefly of a circle) known as sector and segment.


CIRCUMFERENCE & AREA OF A CIRCLE

- $\frac{Circumference}{Diameter} = \pi$

$\Rightarrow \frac{c}{d} = \pi$

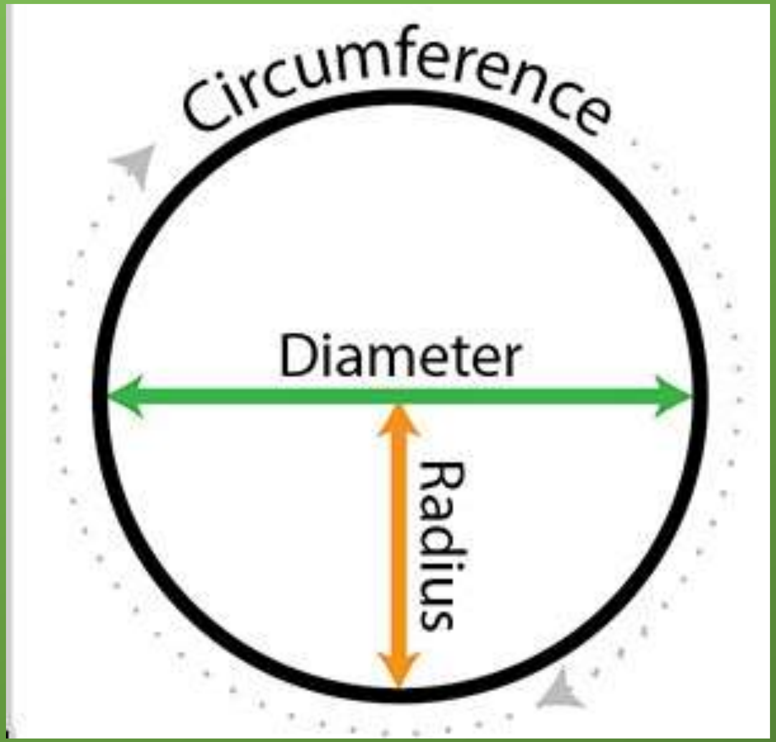
$\Rightarrow c = \pi \times d$

$\Rightarrow c = 2\pi r$

- Area of a circle= πr^2

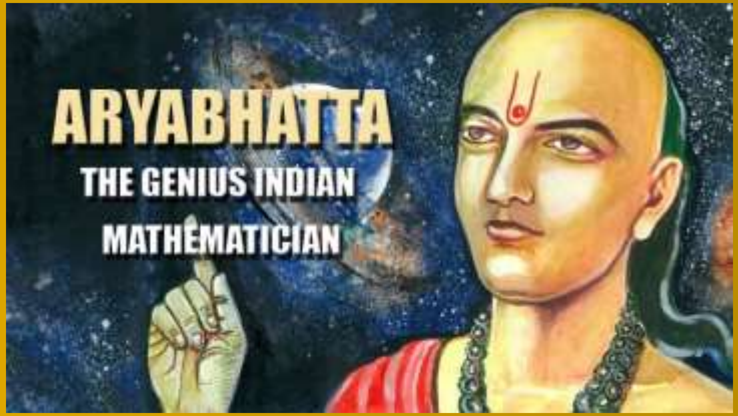
  $= \pi d^2/4$

- Area of a semicircle= $\pi r^2/2$
- Area of a quadrant = $\pi r^2/4$



VALUE OF π

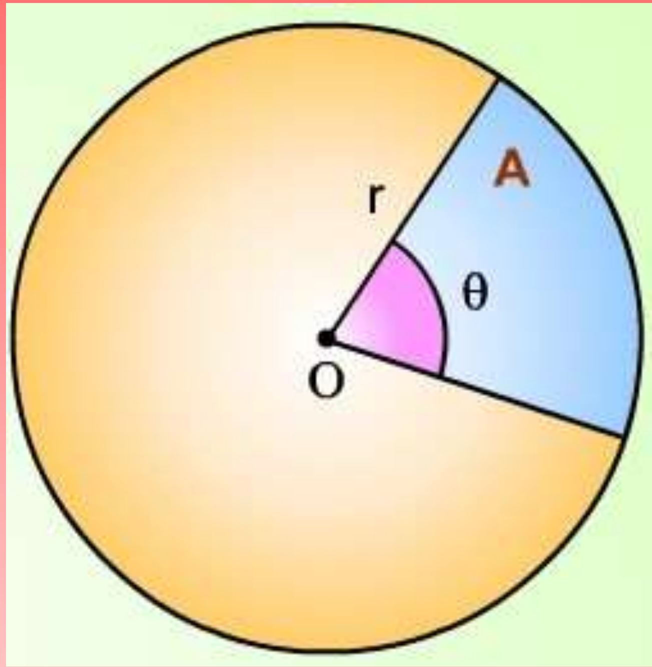
- The great Indian mathematician Aryabhatta gave an approximate value of π which is nearly equal to 3.1416.



- It is also interesting to note that using an identity of the great mathematical genius Srinivas Ramanujan of India, mathematicians have been able to calculate the value of π correct to million places of decimals . we generally take $\pi = 22/7$ or 3.14 approximately

SECTOR

- **Sector:** A sector is a region bounded by 2 radii and an arc.
- **Area of a sector of angle 360^0 = πr^2 .So, area of a sector of angle 1^0 = $\pi r^2/360^0$.**
- **Hence, area of a sector of angle θ = $(\pi r^2\theta)/360^0$**



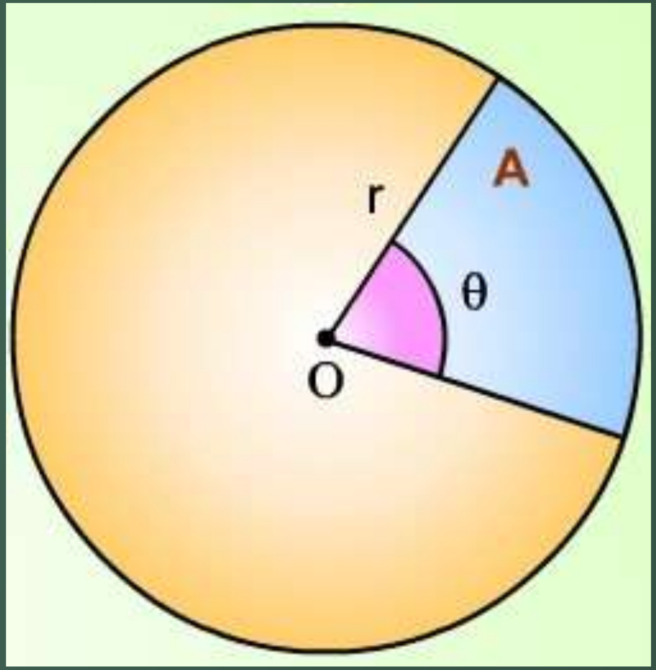
Arc of a Sector

Circumference of a circle of radius r is 2πr.

Length of the arc of a sector of angle 360^0 = 2πr

So, length of the arc of a sector of angle $1^0 = 2\pi r/360^0$

Hence, length of the arc of a sector of angle $\theta = \frac{\theta}{360^0} \times 2\pi r$

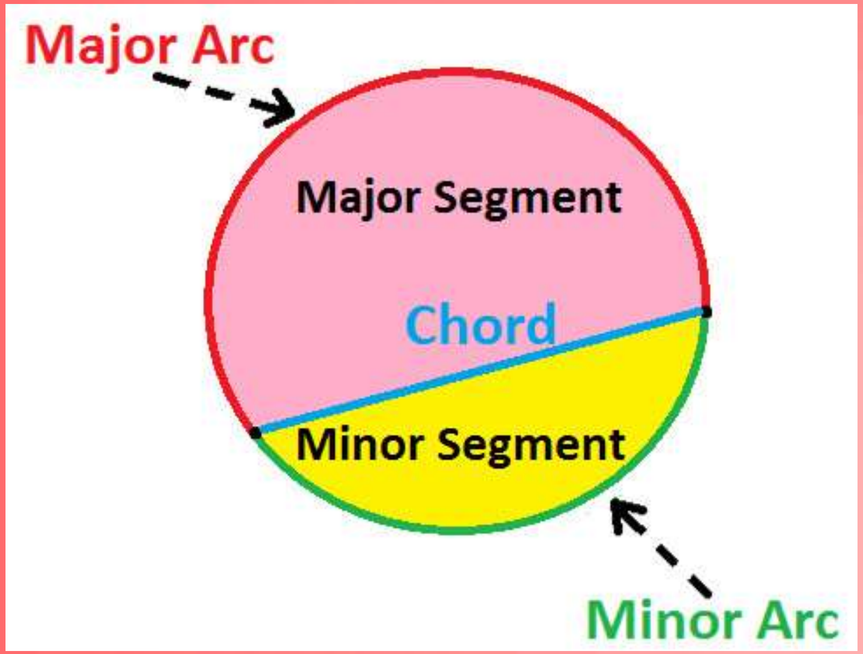


SEGMENT

- A chord of a circle divides the circular region into two parts. Each part is called a segment of the circle.
- There are two parts of area of segment :-

i) Major Segment

ii) Minor Segment

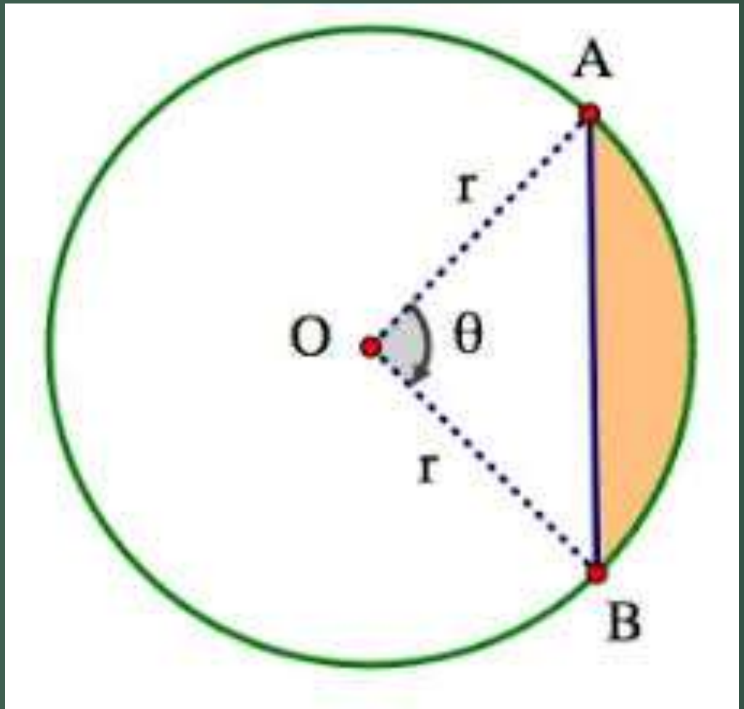


AREA OF SEGMENT

- **Segment**: A segment is a region bounded by a chord and its corresponding arc.
- The area of a segment is equal to the area of the sector - the area of the triangle.

Area of minor segment = $\frac{\theta}{360^0} \times \pi r^2 - \frac{r^2 \sin\theta}{2}$

Area of major segment = area of the circle – area of the minor segment



Some Solved Examples

Q1. **The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Re. 0.50 per m^2. Find the cost of ploughing the field (Take π = 22/7).**

Solution : Length of the fence (in metres) = Total cost/Rate = 5280/24 = 220

So, the circumference of the field = 220 m

If r metres is the radius of the field, then 2πr = 220

$$2 \times (22/7) \times r = 220$$

$$r = (220 \times 7)/ (2 \times 22)$$

$$r = 35$$

Hence, the radius of the field = 35 m

Area of the field = πr^2

= (22/7) × 35 × 35

= 22 × 5 × 35 m^2

= 3850 sq.m.

Cost of ploughing 1 m^2 of the field = Re. 0.50

So, total cost of ploughing the field = 3850 × Re. 0.50 = Rs. 1925

Q2. **Given figure depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.**

Solution : The radius of 1st circle, $r_1 = 21/2$ cm (as diameter D is given as 21 cm)

So, area of gold region = $\pi r_1^2 = \pi(10.5)^2 = 346.5\ cm^2$

Now, it is given that each of the other bands is 10.5 cm wide,

So, the radius of 2nd circle, r_2 = 10.5cm+10.5cm = 21 cm

Thus,∴ Area of red region = Area of 2nd circle − Area of gold region = $(\pi r_2^2 - 346.5)\ cm^2$

$= (\pi(21)^2 - 346.5)\ cm^2 = 1386 - 346.5$

$= 1039.5\ cm^2$

Similarly,

The radius of 3rd circle, r_3 = 21 cm+10.5 cm = 31.5 cm

The radius of 4th circle, r_4 = 31.5 cm+10.5 cm = 42 cm

The Radius of 5th circle, r_5 = 42 cm+10.5 cm = 52.5 cm

For the area of nth regionA = Area of circle n – Area of circle (n-1)

∴ Area of blue region (n=3) = Area of third circle – Area of second circle

$= \pi(31.5)^2 - 1386\ cm^2 = 3118.5 - 1386\ cm^2 = 1732.5\ cm^2$

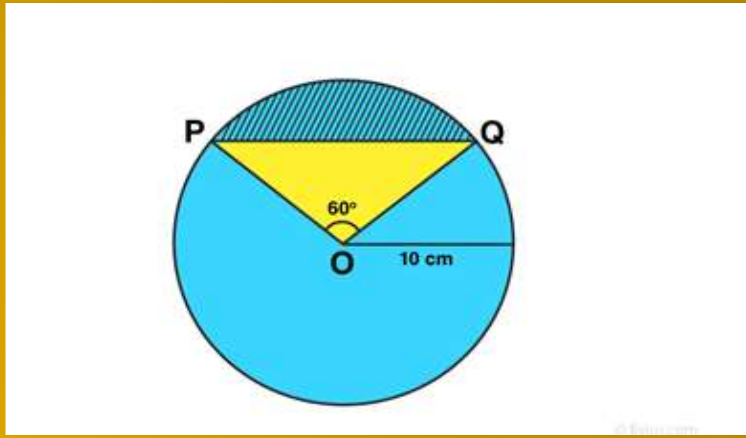
∴ Area of black region (n=4) = Area of fourth circle – Area of third circle= $\pi(42)^2 - 1386\ cm^2$

$= 5544 - 3118.5\ cm^2 = 2425.5\ cm^2$

∴ Area of white region (n=5) = Area of fifth circle – Area of fourth circle= $\pi(52.5)^2 - 5544\ cm^2$

$= 8662.5 - 5544\ cm^2 = 3118.5\ cm^2$

Q3. **A chord PQ of the circle of radius 10 cm subtends an angle of 60 degrees at the centre of the circle. Find the area of minor and major segment of the circle.**



Solution : The radius of the circle = 10 cm.

Chord PQ subtends an angle= 60 degrees at the centre.

PQO is an equilateral triangle, O being the centre of the circle.

Area of minor segment = θ/360 x π r^2 – $\frac{1}{2}r^2\sin\theta$

= $\frac{60^0}{360^0}$ x $\frac{22}{7}$ x 10 x 10 – $\frac{1}{2}$ x 10 x 10 x $\frac{\sqrt{3}}{2}$ = 9.03

Area of circle = π r^2 = $\frac{22}{7}$ x 10 x 10 = 314 (approx.)

Area of major segment = Area of circle – Area of minor segment

= 314 – 9.03

= 304.97 cm^2 (approx.)

Q4. **In a circle of radius 10.5 cm, the minor arc is one-fifth of the major arc. Find the area of the sector corresponding to the major arc.**

Solution : Radius of circle = 10.5 cm

Let x cm be the major arc, then $x/5$ cm be the length of minor arc.

Circumference of circle = $x + \frac{x}{5} = \frac{6x}{5}$ cm

We know, Circumference of circle = 2π r = 2 x $\frac{22}{7}$ x 10.5

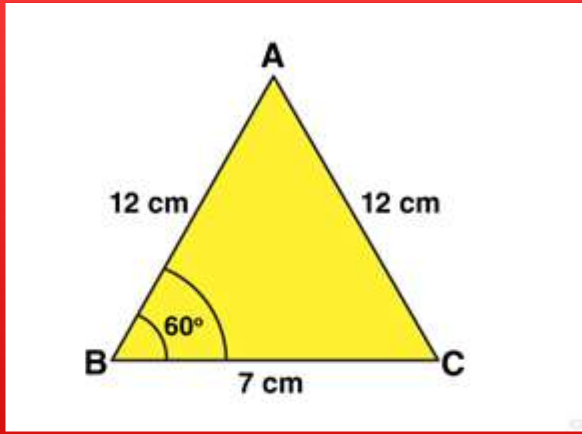
This implies, $\frac{6x}{5}$ = 2 x $\frac{22}{7}$ x 10.5

x = 55 cm

Area of major sector = $\frac{1}{2}$ x 55 x 10.5 = 288.75 (A=$\frac{1}{2}$ x x r)

The area of major sector is 288.75 cm^2.

Q5. **A horse is tethered to one corner of a field which is in the shape of an equilateral triangle of side 12 m. If the length of the rope is 7 m, find the area of the field which the horse cannot graze. [take √3 = 1.73]. Write the answer correct to 2 places of decimal.**



Solution : Each angle of equilateral triangle = 60^0

Side of an equilateral triangle = 12 m

Length of the rope = 7 m

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} x 12 x 12 = 62.35 m^2$

Area of sector with radius 7 m = $\frac{60^0}{360^0} x \frac{22}{7} x 7 x 7 = 25.66\ m^2$

Now, Area which cannot be grazed by horse = Area of an equilateral triangle – Area of sector with radius 7 m

$= 62.35\ m^2 - 25.66\ m^2$

$= 36.68\ m^2$


THANK YOU